The Inverse Problem for Biaxial Materials

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Abstract—**Theory and measurements for the determination of the constitutive parameters of an anisotropic material are described, when a slab of the material is inserted in a rectangular waveguide. If both $\underline{\underline{\epsilon}}$ and $\underline{\underline{\mu}}$ tensors have zero off-diagonal elements (biaxial material), then the six diagonal elements can be determined by measuring amplitude and phase of reflection and transmission coefficients. If the material is nondispersive, two sets of measurements at two different frequencies are sufficient, under TE_{10} excitation. In the more general case of a lossy and dispersive material, two sets of measurements at the same frequency under TE_{10} and TE_{20} excitations are needed. An experimental setup for the latter case is described.**

I. Introduction

In RECENT YEARS, a variety of anisotropic materials have found increasingly numerous and important applications at microwave frequencies. Aside from plasmas and ferrites, which have been studied and used for some time, new materials include fibers with preferred orientation in composites, certain ceramics, and honeycomb structures. Among the applications are antenna radomes, substrates for microstrip antennas and integrated optical devices, and certain types of radar absorbers.

While the electromagnetic theory of anisotropic materials is well established (see, for example, the book by Kong [1]), work still needs to be done on experimental techniques for the determination of the constitutive parameters of these materials. This is a typical inverse problem, in which we must ascertain firstly what measured data are sufficient (and preferably also necessary) for the unequivocal determination of the constitutive parameters, and secondly what experimental setups are preferable to collect the needed data. The problem is complicated by the fact that, in general, the material is both lossy and dispersive. The measurements may be performed either in free space, such as in an anechoic chamber, or inside a waveguide or resonant cavity.

A general treatment of fields in anisotropic guides was first given by Bresler [2]. Specific applications to gyromagnetic materials such as ferrites are found in Kales [3], Barzilai and Gerosa [4], and in the book by Lax and Button [5], among others, while an up-to-date treatment of gyrotropic guides is provided in the book by Hlawiczka [6]. The case of a guide filled with uniaxial material was first studied by Kong and Cheng [7]. The problem of a dielectric uniaxial guide has been reexamined recently by Paul and Shevgaonkar [8], whose work contains a good bibliography on this subject. A rectangular guide filled with a semiconductor in the presence of an external transverse magnetic field (Hall effect) was studied by Engineer and Nag [9], who examined in detail the particular case when the diagonal elements of the complex permittivity tensor are equal. A guide filled with biaxial material was briefly studied by Goncharenko [10] who, however, neglected an important category of possible modes.

In this paper, we consider the inverse problem for an important category of anisotropic materials: biaxial media, for which the relative permittivity and permeability tensors $\underline{\underline{\epsilon}}$ and $\underline{\underline{\mu}}$ are represented by diagonal matrices in a rectangular Cartesian reference system (x, y, z)

$$\underline{\underline{\epsilon}} = \begin{pmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}_{xyz}, \quad \underline{\underline{\mu}} = \begin{pmatrix} \mu_1 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}_{xyz}. \quad (1)$$

The six constitutive parameters ϵ_l and μ_l $(l = 1,2,3)$ are dimensionless numbers and are, in general, complex and frequency-dependent. We seek their determination from measurements of reflection and transmission coefficients for a biaxial sample inserted in a rectangular waveguide.

The boundary-value problem is discussed in Section II, where it is proven that, contrary to a previous statement [10], hybrid modes are not needed and, in fact, a single TE_{m0} mode satisfies the boundary conditions. The general inverse problem is solved in Section III, and the explicit determination of the constitutive parameters is effected in Section IV for a lossless nondispersive material, and in Section V for the more practical case of a lossy dispersive material. For this latter case, a measurement setup involving a network analyzer and a microwave junction which allows for the separate, independent excitation of the sample by either a TE_{10} or a TE_{20} mode is described in Section VI.

II. The Boundary-Value Problem

Consider a metallic rectangular waveguide oriented along the z axis, with horizontal walls of width a parallel to the x axis and vertical walls of height $b \leqslant a$ parallel to the y axis.

Manuscript received June 24, 1983; revised November 21, 1983.

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It is shown below that TE_{m0} modes can exist, when the guide is filled with biaxial material whose principal axes coincide with the (x, y, z) axes of the guide.

With time dependence $\exp(+j\omega t)$, Maxwell's equations are

$$\nabla \times \underline{H} = j\omega\epsilon_0 \underline{\underline{\epsilon}}\,\underline{E}, \quad \nabla \times \underline{E} = -j\omega\mu_0 \underline{\underline{\mu}}\,\underline{H} \tag{2}$$

with $\underline{\underline{\epsilon}}$ and $\underline{\underline{\mu}}$ given by (1). If $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the wavenumber in free space, and if $\underline{E}$ and $\underline{H}$ for a given mode depend on z via the factor $\exp(-\gamma z)$, then the transverse field components are

$$E_x = \frac{-1}{\gamma^2 + k_0^2\epsilon_1\mu_2}\left[\gamma\frac{\partial E_z}{\partial x} + j\omega\mu_0\mu_2\frac{\partial H_z}{\partial y}\right] \tag{3}$$

$$E_y = \frac{-1}{\gamma^2 + k_0^2\mu_1\epsilon_2}\left[\gamma\frac{\partial E_z}{\partial y} - j\omega\mu_0\mu_1\frac{\partial H_z}{\partial x}\right] \tag{4}$$

$$H_x = \frac{-1}{\gamma^2 + k_0^2\mu_1\epsilon_2}\left[-j\omega\epsilon_0\epsilon_2\frac{\partial E_z}{\partial y} + \gamma\frac{\partial H_z}{\partial x}\right] \tag{5}$$

$$H_y = \frac{-1}{\gamma^2 + k_0^2\epsilon_1\mu_2}\left[j\omega\epsilon_0\epsilon_1\frac{\partial E_z}{\partial x} + \gamma\frac{\partial H_z}{\partial y}\right] \tag{6}$$

and the longitudinal components satisfy the coupled equations

$$\left(\frac{\epsilon_1}{\gamma^2 + k_0^2\epsilon_1\mu_2}\frac{\partial^2}{\partial x^2} + \frac{\epsilon_2}{\gamma^2 + k_0^2\mu_1\epsilon_2}\frac{\partial^2}{\partial y^2} + \epsilon_3\right)E_z = \frac{-\gamma}{j\omega\epsilon_0}\left(\frac{1}{\gamma^2 + k_0^2\epsilon_1\mu_2} - \frac{1}{\gamma^2 + k_0^2\mu_1\epsilon_2}\right)\frac{\partial^2 H_z}{\partial x\,\partial y} \tag{7}$$

$$\left(\frac{\mu_1}{\gamma^2 + k_0^2\mu_1\epsilon_2}\frac{\partial^2}{\partial x^2} + \frac{\mu_2}{\gamma^2 + k_0^2\epsilon_1\mu_2}\frac{\partial^2}{\partial y^2} + \mu_3\right)H_z = \frac{\gamma}{j\omega\mu_0}\left(\frac{1}{\gamma^2 + k_0^2\mu_1\epsilon_2} - \frac{1}{\gamma^2 + k_0^2\epsilon_1\mu_2}\right)\frac{\partial^2 E_z}{\partial x\,\partial y} \tag{8}$$

so that, in general, a hybrid mode is needed. However, decoupling of E_z from H_z occurs in two particular cases.

In the first case

$$\epsilon_1\mu_2 = \epsilon_2\mu_1 \tag{9}$$

and (7) and (8) become

$$\left(\epsilon_1\frac{\partial^2}{\partial x^2} + \epsilon_2\frac{\partial^2}{\partial y^2} + \epsilon_3 h^2\right)E_z = 0 \tag{10}$$

$$\left(\mu_1\frac{\partial^2}{\partial x^2} + \mu_2\frac{\partial^2}{\partial y^2} + \mu_3 h^2\right)H_z = 0 \tag{11}$$

where

$$h^2 = \gamma^2 + k_0^2\epsilon_1\mu_2. \tag{12}$$

The field is a superposition of TE and TM modes; in general, condition (9) is not satisfied, and therefore we do not study this case any further.

In the second case, decoupling occurs if either

$$\frac{\partial}{\partial x} \equiv 0 \tag{13}$$

or

$$\frac{\partial}{\partial y} \equiv 0. \tag{14}$$

These two possibilities are essentially one and the same, because we may go from (13) to (14) by a right-angle rotation of the coordinate system about the z axis. We consider only (14) which, incidentally, is satisfied by the dominant TE_{10} mode in the guide filled with air.

Conditions (13) and (14) were neglected in the analysis by Goncharenko [10]; therefore, his statement that TM and TE modes can be supported separately only in uniaxial media ($\epsilon_1 = \epsilon_2$ and $\mu_1 = \mu_2$) is incorrect. In fact, uniaxial media are a particular case of condition (9).

Under condition (14), (3)–(8) and the boundary conditions yield TM modes with identically zero field components, and TE_{m0} modes for which

$$\left.\begin{aligned} H_z &= A_m\cos\left(\frac{m\pi}{a}x\right)e^{-\gamma_m z}\\ E_y &= -\frac{j\omega\mu_0\mu_3 a}{m\pi}A_m\sin\left(\frac{m\pi}{a}x\right)e^{-\gamma_m z}\\ H_x &= \frac{\mu_1\gamma_m a}{\mu_3 m\pi}A_m\sin\left(\frac{m\pi}{a}x\right)e^{-\gamma_m z}\\ E_x &= E_z = H_y = 0, \qquad (m = 1, 2, 3, \cdots)\end{aligned}\right\} \tag{15}$$

where

$$\gamma_m = jk_0\sqrt{\frac{\mu_1}{\mu_3}}\sqrt{\epsilon_2\mu_3 - \left(\frac{m\pi}{k_0 a}\right)^2} = j\beta_m. \tag{16}$$

Note that γ_m depends on ϵ_2, μ_1, and μ_3, and is independent of ϵ_1, ϵ_3, and μ_2. If $\epsilon_2\mu_3$ is real positive, then γ_m is pure imaginary at all operating frequencies above the cutoff frequency

$$f_c = m\frac{c_0}{2a\sqrt{\epsilon_2\mu_3}} \tag{17}$$

where c_0 is the velocity of light in free space.

When the waveguide section $0 \leqslant z \leqslant L$ is filled with a sample of the biaxial material, and the incident TE_{m0} mode with

$$H_z^i = \cos\left(\frac{m\pi}{a}x\right)e^{-j\beta_{0m}z} \tag{18}$$

$$\beta_{0m} = \sqrt{k_0^2 - \left(\frac{m\pi}{a}\right)^2} \tag{19}$$

exists in $z \leqslant 0$, it produces a reflected TE_{m0} mode in $z \leqslant 0$ with

$$H_z^r = R_m\cos\left(\frac{m\pi}{a}x\right)e^{j\beta_{0m}z} \tag{20}$$

and a transmitted TE_{m0} mode in $z \geqslant L$ with

$$H_z^t = T_m\cos\left(\frac{m\pi}{a}x\right)e^{-j\beta_{0m}(z-L)} \tag{21}$$

where we assume that the termination load at $z > L$ is matched. The TE_{m0} field inside the sample $(0 \leqslant z \leqslant L)$ is the combination of two fields, such as (15), propagating in opposite directions. Imposition of the boundary conditions at $z=0$ and $z=L$ yields the following expressions for the reflection coefficient R_m and the transmission coefficient T_m:

$$\frac{1}{T_m}=\cos\beta_m L+\frac{j}{2}\left(u+\frac{1}{u}\right)\sin\beta_m L \tag{22}$$

$$\frac{R_m}{T_m}=-\frac{j}{2}\left(u-\frac{1}{u}\right)\sin\beta_m L \tag{23}$$

where

$$u=\frac{\mu_1\beta_m}{\mu_3^2\beta_{0m}}. \tag{24}$$

III. The Inverse Problem

The inverse problem consists in finding u and β_m from (22) and (23), when L is given and amplitude and phase of R_m and T_m have been measured. Once u and β_m are known, the constitutive parameters are found easily.

In general, the parameters ϵ_2, μ_1, and μ_3 are complex, and therefore β_m and u are also complex. Adding (22) to (23) and solving for u

$$u=\frac{j\sin\beta_m L}{\frac{1}{T_m}+\frac{R_m}{T_m}-\cos\beta_m L}. \tag{25}$$

Subtracting (23) from (22) and solving for u

$$u=\frac{\frac{1}{T_m}-\frac{R_m}{T_m}-\cos\beta_m L}{j\sin\beta_m L}. \tag{26}$$

Now equate (25) to (26), obtaining

$$\cos\beta_m L=\frac{T_m^2-R_m^2+1}{2T_m}. \tag{27}$$

Let us separate real and imaginary parts in the right-hand side of (27) by letting

$$\frac{T_m^2-R_m^2+1}{2T_m}=\alpha'+j\alpha'' \tag{28}$$

where α' and α'' are real numbers, known from measurements. Now let

$$\beta_m=\beta_m'-j\beta_m'' \tag{29}$$

where β_m' and β_m'' are real quantities such that

$$\beta_m'>0, \qquad \beta_m''\geqslant 0. \tag{30}$$

The second part of (30) implies that the biaxial medium is lossy (in the case of an active medium, we would have $\beta_m''\leqslant 0$, and the discussion would proceed in a manner similar to the lossy case). Now (27) yields

$$\begin{cases}\cos\beta_m' L\cosh\beta_m'' L=\alpha'\\ \sin\beta_m' L\sinh\beta_m'' L=\alpha''\end{cases} \tag{31}$$

and our inverse problem is reduced to finding β_m' and β_m'' from the system (31), for given α' and α'' and under restrictions (30). Once β_m is known, u is given by either (25) or (26).

It is seen from (31) that $\cos\beta_m' L$ has the sign of α' and, because of (30), $\sin\beta_m' L$ has the sign of α''

$$\operatorname{sign}(\cos\beta_m' L)=\operatorname{sign}\alpha', \operatorname{sign}(\sin\beta_m' L)=\operatorname{sign}\alpha''. \tag{32}$$

Elimination of the hyperbolic functions from (31) yields

$$\cos^2\beta_m' L=\xi=\frac{1}{2}\left[\alpha'^2+\alpha''^2+1-\sqrt{(\alpha'^2+\alpha''^2+1)^2-4\alpha'^2}\right] \tag{33}$$

and therefore from (32)

$$\cos\beta_m' L=\sqrt{\xi}\ \operatorname{sign}\alpha', \sin\beta_m' L=\sqrt{1-\xi}\ \operatorname{sign}\alpha''. \tag{34}$$

From (34) we have

$$\beta_m'=\tilde{\beta}_m'+\frac{2\pi n}{L}, \qquad n=0,1,\cdots \tag{35}$$

where $\tilde{\beta}_m'$ is known, and $0<\tilde{\beta}_m' L\leqslant 2\pi$. Elimination of the trigonometric functions from (31) and use of (33) yields

$$\sinh\beta_m'' L=\sqrt{\alpha'^2+\alpha''^2-\xi} \tag{36}$$

from which β_m'' is uniquely determined. Thus, β_m is determined, aside from the choice of the integer n in (35). If the length L of the sample is sufficiently small, then $n=0$; however, in many practical cases the length L cannot be arbitrarily chosen; then, n is uniquely determined from nondispersive media by carrying out measurements at two different frequencies, as explained in the following section. If the material is dispersive and the sample is not sufficiently thin, a rough preliminary estimate of the values of the constitutive parameters still allows us to determine n for a given L.

IV. Lossless Nondispersive Material

If the material of the sample is lossless and nondispersive, the constitutive parameters may be determined by taking measurements at two different operating frequencies ω and $\omega^{(1)}$ under the same TE_{m0} mode (in practice, the dominant TE_{10} mode).

Let β_m and u be determined as indicated in the previous section for frequency ω, and let $\beta_m^{(1)}$ and $u^{(1)}$ be the corresponding values at frequency $\omega^{(1)}$. From (24)

$$\beta_m=\frac{\mu_3^2}{\mu_1}\beta_{0m}u, \quad \beta_m^{(1)}=\frac{\mu_3^2}{\mu_1}\beta_{0m}^{(1)}u^{(1)} \tag{37}$$

whereas from (35) and the lossless properties of the sample

$$\beta_m=\tilde{\beta}_m+\frac{2\pi n}{L}, \quad \beta_m^{(1)}=\tilde{\beta}_m^{(1)}+\frac{2\pi n}{L} \tag{38}$$

where n is the same integer in both (38), since $\tilde{\beta}_m\to\tilde{\beta}_m^{(1)}$ and $\beta_m\to\beta_m^{(1)}$ when $\omega\to\omega^{(1)}$. To determine n, we equate the ratio of the two equations (37) to the ratio of the two equations (38), obtaining

$$n=\frac{L}{2\pi}\cdot\frac{\frac{\beta_{0m}u}{\beta_{0m}^{(1)}u^{(1)}}\tilde{\beta}_m^{(1)}-\tilde{\beta}_m}{1-\frac{\beta_{0m}u}{\beta_{0m}^{(1)}u^{(1)}}}. \tag{39}$$

By equating the ratio between (16) and the corresponding equation for $\beta_m^{(1)}$ to the ratio of the two equations (37), we obtain

$$\epsilon_2\mu_3 = \left(\frac{m\pi}{k_0 a}\right)^2 \cdot \frac{1-\dfrac{\beta_{0m}^2 u^2}{\beta_{0m}^{(1)2} u^{(1)2}}}{1-\dfrac{\beta_{0m}^2 u^2 k_0^{(1)2}}{\beta_{0m}^{(1)2} u^{(1)2} k_0^2}} \tag{40}$$

and therefore the product $\epsilon_2\mu_3$ is known. Now the ratio μ_1/μ_3 is obtained from (16), and the ratio μ_3^2/μ_1 from either of (37); hence, both μ_1 and μ_3 are known, and ϵ_2 is derived from the knowledge of the product $\epsilon_2\mu_3$.

In conclusion, if ϵ_2, μ_1, and μ_3 are real and independent of frequency, they can be found by measuring R_m and T_m at two different frequencies for the dominant TE_{10} mode. By changing the orientation of the sample in the waveguide, the other three constitutive parameters are similarly determined.

V. Lossy Dispersive Material

If the medium is dispersive, measured data at different frequencies cannot be mixed together, and the only way to obtain a second, independent measurement of R_m and T_m is to excite the guide with a different mode. Thus, we conduct two separate measurements at the same frequency ω, the first with the TE_{10} mode yielding R_1 and T_1, the second with the TE_{20} mode yielding R_2 and T_2. From (16)

$$\beta_1 = \sqrt{\frac{\mu_1}{\mu_3}}\sqrt{k_0^2\epsilon_2\mu_3 - \left(\frac{\pi}{a}\right)^2}$$
$$\beta_2 = \sqrt{\frac{\mu_1}{\mu_3}}\sqrt{k_0^2\epsilon_2\mu_3 - 4\left(\frac{\pi}{a}\right)^2} \tag{41}$$

and from (24)

$$\beta_1 = \frac{\mu_3^2}{\mu_1}\beta_{01}u_1, \quad \beta_2 = \frac{\mu_3^2}{\mu_1}\beta_{02}u_2 \tag{42}$$

where $u_{1,2}$ are the values of u measured with TE_{10}, TE_{20} excitation, respectively, and

$$\beta_{01} = \sqrt{k_0^2 - \left(\frac{\pi}{a}\right)^2}, \qquad \beta_{02} = \sqrt{k_0^2 - 4\left(\frac{\pi}{a}\right)^2}. \tag{43}$$

Equating the ratio of (41) to the ratio of (42) and solving for the product $\epsilon_2\mu_3$, we obtain

$$\epsilon_2\mu_3 = \frac{1-4\left(\dfrac{\beta_{01}\mu_1}{\beta_{02}\mu_2}\right)^2}{1-\left(\dfrac{\beta_{01}\mu_1}{\beta_{02}\mu_2}\right)^2}\left(\frac{\pi}{k_0 a}\right)^2. \tag{44}$$

Equating the first of (41) to the first of (42) and solving

$$\frac{\mu_1^3}{\mu_3^5} = \frac{\beta_{01}^2 u_1^2}{k_0^2\epsilon_2\mu_3 - \left(\dfrac{\pi}{a}\right)^2}. \tag{45}$$

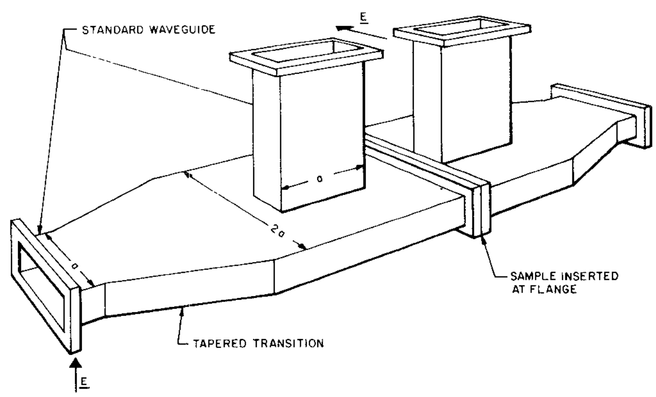

Fig 1. Sketch of TE_{10}–TE_{20} measurement fixture.

If the sample is sufficiently thin, $\beta_1 L < 2\pi$. Let us assume that this is the case, then $\beta_1 = \tilde{\beta}_1$ and from (41)

$$\frac{\mu_1}{\mu_3} = \frac{\tilde{\beta}_1^2}{k_0^2\epsilon_2\mu_3 - \left(\dfrac{\pi}{a}\right)^2}. \tag{46}$$

Now μ_1 and μ_3 are obtained at once from (45) and (46), then ϵ_2 is given by (44). Similar measurements with a rotated sample yield ϵ_1, ϵ_3, and μ_2.

Of course, the method described in this section is applicable also to the simpler case of lossless nondispersive material considered in Section IV.

VI. Measurement Procedure

A waveguide fixture that is convenient for exciting TE_{10} and TE_{20} modes has been suggested in the literature [11] and is shown in a form adaptable for use with modern network analyzers in Fig. 1. The TE_{10} mode is excited by the horizontal arms that are standard waveguides beyond the tapered transitions. The TE_{20} mode is excited by the branch arms that also are standard waveguides. Both modes can propagate in the central part of the fixture which is of double-width. Beyond the transition region the horizontal arms are below cutoff for TE_{20} modes. With reasonable precision in fabrication of the fixture, TE_{10} modes will not be coupled into the branch arms. Hence, a circuit through the branch arms can be used to measure reflection and transmission coefficients due to the TE_{20} mode, while a circuit through the side arms can be used to measure reflection and transmission coefficients for the TE_{10} mode. Clearly, impedance matching will be required at the branch arms and at the tapered transitions.

Equations (27) and (41)–(46) require measurements of four complex quantities to determine three unknown complex components of the constitutive parameters from each orientation of the sample. The additional measured quantity greatly simplifies the mathematical steps that are required to solve the inverse problem. In addition, if three orientations of the sample are used to determine the six unknowns, the measurements provide ample redundancies that aid in immediately evaluating the quality of the measurement.

VII. Discussion and Conclusion

In the previous analysis, the only point that requires special care is the determination of the integer n in (35). However, this value is easily found for nonmagnetic ($\mu_1 = \mu_2 = \mu_3$) materials; from (34), $\beta'_m = \beta_{0m}$ Reu, and thus n is found from (35).

Rather than measuring reflection and transmission coefficients in a guide with a matched load, we may inquire as to whether measuring the reflection coefficient $\tilde{R}_m$ for a sample backed against a short is sufficient to solve the inverse problem. The answer is negative, as is seen in the simple case of a lossless sample with a metal wall at $z = L$. Then $|\tilde{R}_m| = 1$ and only $\angle\tilde{R}_m$ carries useful information. We find that

$$\frac{\mu_1 \beta_m L}{\mu_3^2 \tan \beta_m L} = \beta_{0m} L \tan\left(\frac{1}{2}\angle\tilde{R}_m\right). \tag{47}$$

Even in the particular case $\mu_1 = \mu_3 = 1$, the transcendental equation (47) has an infinite number of solutions for β_m, each leading to a different value of ϵ_2 via (16).

We have derived an analytical solution to the inverse problem of determining the constitutive parameters of a biaxial material, which may be lossy and dispersive, from measurements of reflection and transmission coefficients in a rectangular waveguide containing a sample of the material. We have also suggested an experimental setup to perform the needed measurements.

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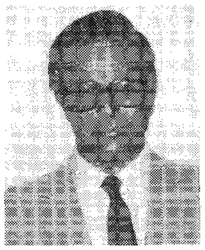

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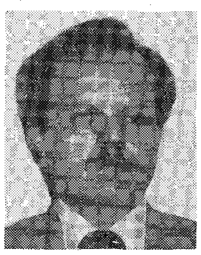

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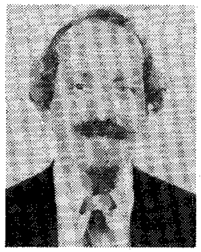

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Perturbation Analysis and Design Equations for Open- and Closed-Ring Microstrip Resonators

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Abstract —**Simple closed-form expressions for the resonant frequency and electromagnetic field distribution for various modes of the open- and closed-ring microstrip resonators are derived by utilizing the perturbation analysis of the equivalent curved waveguide model. These results are shown to be in good agreement with the exactly computed values obtained by the solution of the eigenvalue equation for the equivalent waveguide model and the experimental data. The effect of gap capacitance on the eigenvalues of the open-ring resonator is also examined.**

I. Introduction

MICROSTRIP annular ring resonators have been used in recent years for various applications including microwave filters and planar antenna elements [1]–[9]. The basic properties of these structures, that is, the resonant frequency and the field distribution for various modes, have been evaluated by utilizing a number of techniques including the numerical solution of the eigenvalue problem associated with the equivalent two-dimensional curved waveguide model [1]–[9]. Closed-form solutions expressing the resonant frequencies and fields in terms of the geometry of the structure (or the corresponding model) are not yet available for the design of such structures except for the simplified case where the effect of the curvature is totally neglected. In this paper, simple closed-form expressions for the resonant frequencies and the electromagnetic fields are derived by utilizing the perturbation analysis of the equivalent curved waveguide [10], [11] with electric and magnetic walls. The accuracy and range of validity of the results are also examined together with the effects of small gap angles on the resonant characteristics of the open-ring structures.

II. Theory

The magnetic wall curved waveguide models for the open- and closed-ring microstrip resonators are shown in Fig. 1. The model is characterized by its effective dimensions and the medium permittivity which are determined from the solution of the corresponding microstripline problem [12], and the inclusion of the effect of curvature on the model [3], [4]. The model assumes that the substrate height h is small ($h \ll \lambda$, the wavelength) and, hence, the fields are constant along the z-direction. The solutions of interest for fields are then the TM modes with respect to the

Manuscript received July 15, 1983; revised October 20, 1983.

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0018-9480/84/0400-0405$01.00 ©1984 IEEE